Quantitative Methods

Internal Rate of Return (IRR) and Net Present Value (NPV)

Internal Rate of Return (IRR) and Net Present Value (NPV)

Learning outcomes……..

- **Calculate and Interpret** NPV and IRR of a series of investment cash flows
- **Explain** how NPVs and IRRs can be used in investment decision-making and their limitations

Internal Rate of Return (IRR) and Net Present Value (NPV)

Net Present Value (NPV)

$PV = FV/(1+r)^T$ → Cost of Capital

- NPV is PV of future revenues less Cost of investment (generally investment happens initially).
- Or in other words NPV = PV of Cash Inflows (Receive) less PV of Cash Outflows (Pay)

Cash inflows and outflows occur at diff. points in time.

- We calculate NPV at a given discount rate, which should be the cost of capital for the firm.

Example:

- What is the NPV of following investment, assuming cost of capital of 12%?

Year	0	1	2	3
Cash Flows	-100	34	57	68

Internal Rate of Return (IRR) and Net Present Value (NPV)

Net Present Value (NPV)

Outflows : – ve sign
Inflows : + ve sign

Example:

- What is the NPV of following investment, assuming cost of capital of 12%?

Year	0	1	2	3
	outflow	inflows	inflows	inflows
Cash Flows	-100	34	57	68

$$NPV = -100 + \frac{34}{(1+12\%)^1} + \frac{57}{(1+12\%)^2} + \frac{68}{(1+12\%)^3}$$

Internal Rate of Return (IRR) and Net Present Value (NPV)

Net Present Value (NPV)

Example:

- What is the NPV of following investment, assuming cost of capital of 10%?

Year 0: -500
Year 1: 215
Year 2: 190
Year 3: 70
Year 4: 130

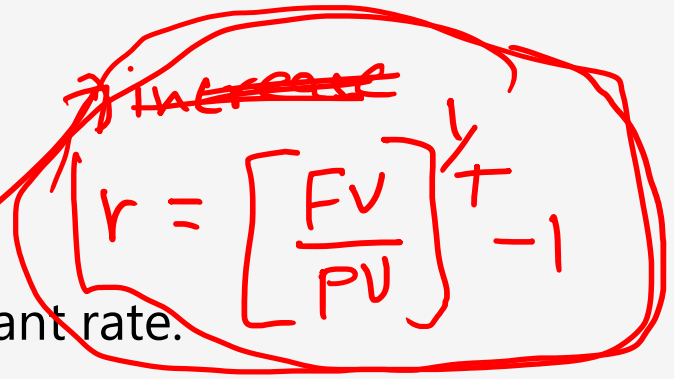
$$NPV = -500 + \frac{215}{1.1} + \frac{190}{1.1^2} + \frac{70}{1.1^3} + \frac{130}{1.1^4}$$

$$= -6.14$$

Internal Rate of Return (IRR) and Net Present Value (NPV)

Net Present Value (NPV)

- NPV is widely used in valuation of financial assets.
- An investment may increase or decrease at a constant rate.
- If it increases at a constant rate, we call it compounded annual growth rate (CAGR).
- The formula to calculate a constant rate of decline in the value of investment is as follows:
- $r = 1 - [FV/PV]^{1/T}$
- Suppose a machine costs $10,000 now. After 3 years, it is likely to cost $6,000. What is the constant rate of decline in machine value?
  - $r = 1 - [6000 / 10000]^{1/3} = 15.7\%$



Internal Rate of Return (IRR) and Net Present Value (NPV)

Net Present Value (NPV)

$$\left[\frac{FV}{PV}\right]^{1/T} - 1$$

1. What is compounded annual growth rate for the following investment?

Money invested today: 11,500 — PV

Money received after 5 years: 16,200 — FV

$$\left[\frac{16200}{11500}\right]^{1/5} - 1 = 7.1\%$$

2. What is the constant rate of change in the following investment?

Machine purchased today for $1,200,000 — PV

Machine value after 7 years: $560,000 — FV

$$\left[\frac{560000}{1200000}\right]^{1/7} - 1$$

$$= -10.32\%$$

Internal Rate of Return (IRR) and Net Present Value (NPV)

NPV → absolute number

Internal Rate of Return (IRR) → Trial & Error

- IRR is a rate at which NPV becomes zero.
- Or, in other words, it is the rate at which PV cash outflows (due to cost of investments) equals PV of cash inflows (due to returns from investments).

Simple Example: 1 year investment

- Suppose a bond costs $92 and returns $100 after 1 year, assuming no other cash inflows, what is the IRR?
- As per the above definition: PV of cash outflows = PV of cash inflows
  - Hence, $92 = 100/(1 + irr)^1$, irr = 100/92 -1 = 8.69%

PV of Cash outflow = PV of Cash inflow

Internal Rate of Return (IRR) and Net Present Value (NPV)

Internal Rate of Return (IRR)

Example: 2 year investment

- Suppose a bond costs $92 and returns $100 after 2 years. It also pays an interest of $4 at the end of each year. What is the irr of investment?
- As per the above definition: PV of cash outflows = PV of cash inflows
  - Hence, $92 = 4/(1+irr)^1 + 104/(1 + irr)^2$, ~~irr = 100/92 - 1~~ = 8.52%
  - We can calculate the irr by trial and error method as follows:
  - Start with one discount rate say 10%. Calculate the value of RHS equation. LHS is 92 in this example. If RHS works out to be lower than 92, then discount rate should be decreased, else it should be increased.
  - We can approximate the irr by 'interpolation' between the two discount rates used.

Internal Rate of Return (IRR) and Net Present Value (NPV)

IRR formula: PV of Cash inflows = PV of Cash outflows → Dis rate at which this happen is IRR

NPV and IRR in decision making

→ Cost of Capital / Discount rate

NPV criteria:

- Accept the project if NPV is positive, reject if it is negative
  - If NPV is positive, irr will be greater than cost of capital and if NPV is negative, irr will be lower than cost of capital

If Cost of Capital → 10%
NPV: +500, IRR > 10%

IRR criteria:

- Accept the project, if IRR > cost of capital, else reject

Conflict in rankings as per NPV and IRR

- If more than 1 projects are under consideration and funding is limited, NPV and IRR can give different rankings. For example, NPV may rank Project X higher than Project Y and IRR may rank Project Y higher than Project X
- If this happens, always **NPV** should be used to take the decision

$$600 = \frac{200}{(1+IRR)^1} + \frac{300}{(1+IRR)^2} + \frac{400}{(1+IRR)^3}$$

Internal Rate of Return (IRR) and Net Present Value (NPV)

NPV and IRR in decision making

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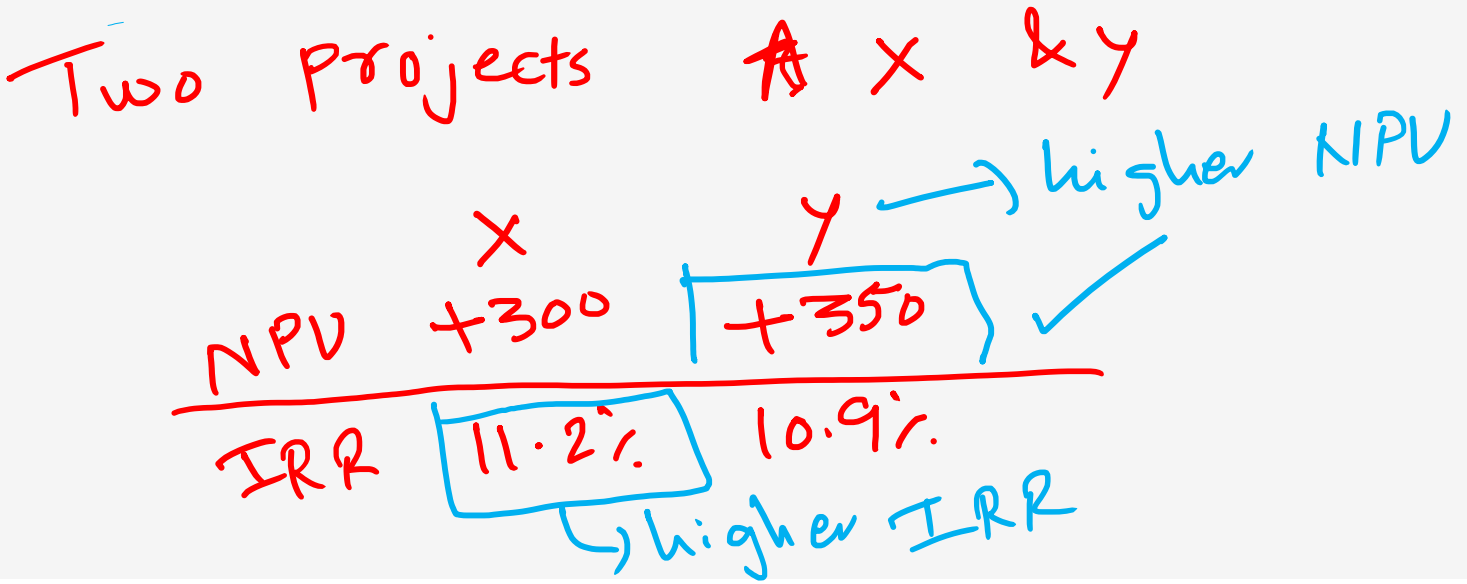

Internal Rate of Return (IRR) and Net Present Value (NPV)

NPV and IRR in decision making

Conflict in rankings as per NPV and IRR

- The reasons why the difference in ranking arises are
  - Projects may require significantly different investments in amounts
  - Cash inflow patterns could be quite different for the projects
  - As mentioned, use NPV in such cases
- In general, NPV is superior to IRR
- But IRR is also popularly used
- One problem with IRR is that for a particular investment there could be 'no irr' or 'multiple irrs'. This happens when the project has cash outflows and inflows keep alternating.