Quantitative Methods

Correlation and Bivariate Linear Regression

Regression Analysis: The least squared method: Scatter diagram

- Thus, $\boxed{\text{error}}$ $e_i = Y_i - \hat{Y}_i = Y_i - (a + bX_i)$ where a and b are chosen to minimize sum of all errors squared as follows:

estimated

Intercept $a = \bar{Y} - b\bar{X}$

$$b = \frac{\text{Covariance}(X, Y)}{\text{Variance}(X)}$$

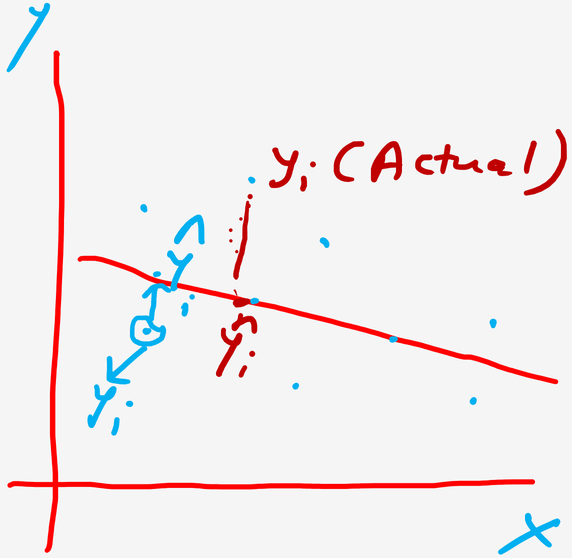
slope

- Slope b can also be calculated as follows:

$$\frac{\sum X_i Y_i - n\bar{X}\bar{Y}}{\sum X_i^2 - n\bar{X}^2}$$

$Y_i \rightarrow$ Actual value

$\hat{Y}_i \rightarrow$ Estimated value (Value on regression line)



$Y_i = a + bX_i$

Correlation and Bivariate Linear Regression

Error = Actual − Estimated   370020 − 350050 =

Regression Analysis: The least squared method: Scatter diagram

- Thus, error $e_i = Y_i - \hat{Y}_i = Y_i - (a + bX_i)$ where a and b are chosen to minimize sum of all errors squared as follows:

$Y_i = a + bX_i$

$$a = \bar{Y} - b\bar{X} \qquad b = \frac{\text{Covariance}(X, Y)}{\text{Variance}(X)}$$

The regression line between sales of company & income levels is as follows. Sales are dependent variable (Y).

- Slope b can also be calculated as follows: $\dfrac{\sum X_iY_i - n\overline{XY}}{\sum X_i^2 - n\bar{X}^2}$

$y = 50 + 70x$

If x income level ~~is~~ is \$5000 what is estimate of y?

$y = 50 + 70 \times 5000$

$y = \underline{350050} \rightarrow \hat{y}$

If actual value of y is 370020, what is error?

how much change in y is due to change in x → R^2

		e^2
e_1	3	9
e_2	−2	4
e_3	4	16
		29

Correlation and Bivariate Linear Regression

e_i → Actual − Estimate

Regression Analysis: The least squared method: Scatter diagram

- Since we know how to calculate error for each value, we can calculate **sum of all residuals squared** or **residual sum of squares** SS_e as follows: $SS_e = \sum e_i^2$ → minimum → line of best fit

- The **total variance** → SD^2 is calculated as SS_y: $SS_y = \sum (Y_i - \bar{Y})^2$

  → total change in y

- The above two values are used to calculate **Coefficient of Determination**, R^2. $R^2 = 1 - \left(\frac{SS_e}{SS_y}\right)$

  → depends upon x

- Coefficient of determination is proportion of variation in dependent variable that **is explained** by the regression model. how much of ~~total~~ total change y is caused due to x

  If R^2 is 'high', regression

- In other words, it reflects how well the regression model fits the observed data.

  equation is good/useful.

Correlation and Bivariate Linear Regression

Regression Analysis: The least squared method: Scatter diagram

- One more relationship is $\underbrace{SS_e}_{\text{errors}} + \underbrace{SS_f}_{\text{predicted}} = \underbrace{SS_y}_{\text{total variation}}$ where SS_f is sum of squared predicted.
- A practical example in financial markets is **market model** in which return on a security is related to market return and a constant.

used extensively in y

stock / share

$\boxed{y} = a + b\boxed{X}$

- $\boxed{Ri} = a + b * \boxed{Rm}$

$SS_y \longrightarrow$ total variation : 120

$SS_e \longrightarrow$ Error : 40

$SS_f \longrightarrow$ Predicting : 80 by regression equation (changes in X)

↳ by X

$$R^2 = 1 - \frac{SS_e}{SS_y}$$

$$R^2 = 1 - \frac{40}{120} = 2/3 \rightarrow 66.7\%$$

$\rightarrow$ 66.7% changes in y are explained of total Δ in y $\rightarrow$ 36% is explained by regression equation

Correlation and Bivariate Linear Regression

Regression Line Explored

$R^2 \rightarrow$ coeff. of determination
$R \rightarrow$ coeff. of Correlation

- Correlation coefficient (r) between actual values of Y_i and fitted values of Y_i determines how well the regression line fits the data.

$r \rightarrow 0.6$, calculate coeff of determination: $r^2 = 0.6^2 \rightarrow 0.36 \rightarrow 36\%$

  - Values of 'r' closer to 1 indicate good fit while values away from 1 indicates poor fit.
  - R^2 is coefficient of determination which is already discussed. Higher R^2 indicates good fit.

GDP, int rates

$y = a + bx$ $\qquad$ $y = a + bx + cz$

  - If we add more independent variables, R^2 goes up. Hence the concept of '**adjusted R^2**' is introduced. when there are more than 1 indep. variable

If R^2 is high $\rightarrow$ R is high $\rightarrow$ Y & X are highly correlated
R^2 is low $\rightarrow$ R " low $\rightarrow$ Y & X are weakly correlated

Correlation and Bivariate Linear Regression

Regression Line Explored

Cause — Effect

Coke sales : Oil price → Meaningless

- Regression can be run between any variables. However, finding such relationships between economic variables does not mean '**causal**' relationship exists.
- '**Data mining**' involves finding empirical relationships between variables for which there is no real causal relationship.
- This can lead to incorrect policy decisions or investment recommendations.
- Data mining involves analysing large amounts of data to identify patterns to exploit any such relationship.

Correlation and Bivariate Linear Regression

Regression Line Explored

Example 1:

- Assume that following regression equation is given: Y = 6 + 2X. Suppose the value of X is forecast to be 20. What will be the value of Y?

Example 2:

R^2 SSe

- Calculate coefficient of determination given sum of squared residuals is 9 and total sum of squared value of Y is 25.

SSy

$$R^2 = 1 - \frac{SSe}{SSy} = 1 - \frac{9}{25} =$$

Correlation and Bivariate Linear Regression

Y Sales X GDP

2016
17
18
19
20
21

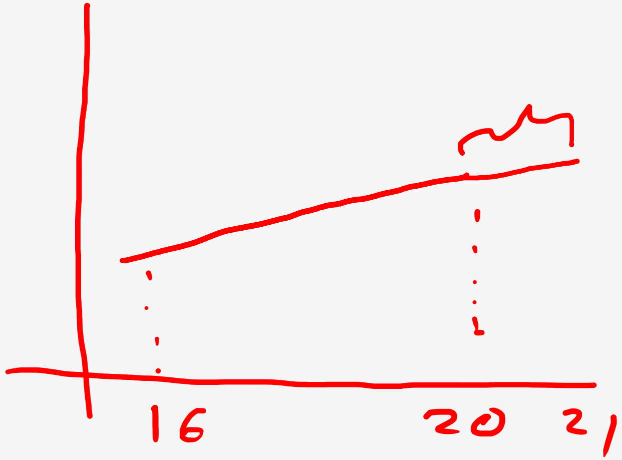
→ relationship

Forecasting, Extrapolation and Interpolation

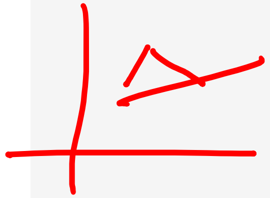
- **Extrapolation** means using the equation to calculate values beyond a particular range. We assume a linear relationship exists while extrapolating.

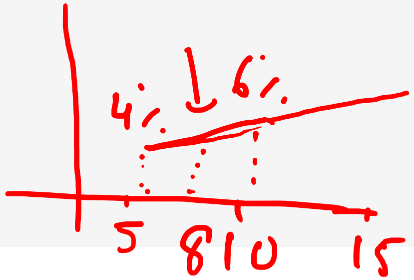
- This could be an incorrect assumption.

- Moreover, stability of coefficients is also an issue, a and b may change.



- **Interpolating** means calculating value in between a range or any two numbers.





Bond → pay int.

Correlation and Bivariate Linear Regression

Ordinary least squares

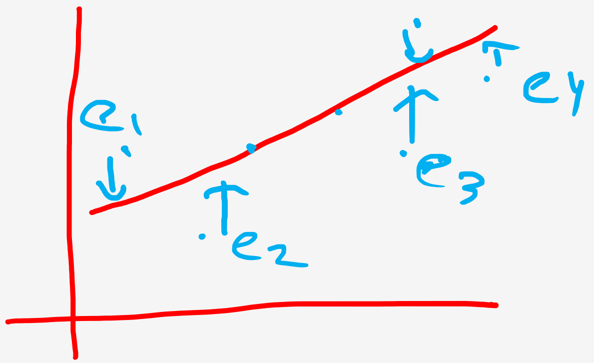
Systematic Omissions and desirable properties of OLS estimators

a & b formulae

- OLS estimator's desirable properties are:
  - It should be Best Linear Unbiased Estimator (BLUE).
  - 1) It should be **unbiased**, meaning the mean of the sampling distribution of that statistic is equal to the parameter being estimated.
  - 2) **Best** refers to lowest variance of estimators
- × As per Gauss-Markov theorem, if equation errors have 1 expectation zero, they are 2 uncorrelated and have 3 equal variances, then BLUE of coefficient is given by OLS estimator.

Correlation and Bivariate Linear Regression

If errors are correlated → biased estimator



Systematic Omissions and desirable properties of OLS estimators

- If the assumptions discussed on earlier slide do not hold, then
  - If the errors are **correlated** (serial correlation), estimator is **not unbiased**.
    biased
    heteroskedasticity
  - If the variance of error terms is not constant, it is called **heteroscadasticity**. If this happens our inferences about coefficients could be incorrect.

  - In forming regression equation, we are also interested in finding out whether we have missed any in dependent variables that should have been included.

predict well

↳ lower R^2 → missed some independent variables.

Index Numbers

Index Numbers

Learning outcomes……..

- **Explain** the role of financial market indices
- **Explain and Calculate** a price relative for a share or index and **Calculate** an index level for the current year, given the base year data and the current year data
- **Calculate** an index level having re-based the index series

Index Numbers

Learning outcomes……continued

- **Calculate** a price weighted index, an equally weighted index, a market value-weighted index and a geometrically weighted index

- **Identify and Explain** the impact of a free float versus market capitalization methodology on index calculation

- **Describe** the composition and construction of key global bond and equity market indices and identify strengths and weaknesses of their respective construction methods

Index Numbers

Use
1. to know change in port. value
2. to evaluate performance of fund mgrs
3. Used as benchmark
4. Analyse market performance

➢ Index numbers are used to represent percentage change in economic variables compared to some base year

Qatar GDP: 2015 → 100 → 2021

Financial Market Indices and Fund Management

➢ In finance, index is created change in ~~change in~~ measure of value of a portfolio which may represent an equity market, bond market or some sector of market. → group of stocks

↳ stocks are trading → Price changing : 2010 → US stocks; 2022 → US stocks

➢ Fund manager is a professional who manages funds on behalf of the others. An index can be used to compare the performance of fund manager compared to the relevant market.

→ standard

➢ Moreover, indices are used as **benchmarks** for variety of purposes such as comparing a security's performance against an index or as mentioned above evaluating a fund manager's performance relative to index.

➢ Investors / fund managers can analyse market performance by studying movements in indices.

US → more than 5000 Companies shares are trading

Investor $1mln → FM → decide which stocks to invest
QIA → mln $
fund mgr → 1 month → 3 month
today

Index Numbers

Financial Market Indices and Fund Management

- Other applications of indices include timing investment decisions by studying the current index level. Creating a portfolio that **mimics** an index.

mirror image → popular in US

- From Jan 2018, new European legislation regulates provision, contribution and use of benchmarks. → indices
- Robust and reliable benchmarks are aimed to be created and hence a common, consistent approach that minimizes conflicts of interest in creating indices is proposed by the legislation.

its value

Definition of Index as per EU Benchmark Regulation (BMR):

- An index is publicly available, regularly determined, either by (1) applying formula or other (2) calculation or making an assessment (3) on the basis of the value of one or more underlying assets or prices (including estimated prices, actual or estimated interest rates, quotes and committed quotes or other values or surveys).

Index Numbers

Financial Market Indices and Fund Management

- An index becomes a benchmark under EU BMR if

  interest payment

  - It is used to determine amount payable under a financial instrument / contract, or the value of financial instrument or
  - It is used to measure the performance of an investment fund for the purpose of tracking the return

    QIA → Mr X → US equity in 2022 → 10% to QIA

    → Benchmark index → 12%

  - Defining the asset allocation or a portfolio or computing performance fee → index

Bonds → pay fixed int rate say 6%

→ pay floating int rate → keeps changing based on benchmark index

→ equity
→ bonds } index

Index Numbers

Index Numbers Construction

- An index number is creating by treating value in the base year as 100 and rescaling / expressing all other values as a proportion of 100.

- **Example**:

Actual Values

$$\text{index} = \frac{\text{Curr yr value}}{\text{Base yr Value}} \times 100$$

INDEX NUMBER CONSTRUCTION

Year	GDP (£ billion)	Index (Year 1 = 100)
Year 1 → base	10.0	100
Year 2	10.2	102
Year 3	11.0	110
Year 4	11.6	116
Year 5	12.0	120

$\frac{10.2}{10} \times 100$

$\frac{11}{10} \times 100$

$\frac{11.6}{10} \times 100$

$\frac{12}{10} \times 100$

- An index number always reflects percentage change in a variable relative to **base year**.

Index Numbers

$\frac{\text{New} - \text{Old}}{\text{Old}}$

$0.00 \quad 00\%$

$0.20 \quad 20\%$

$\frac{\text{New Val}}{\text{Old Val}} - 1$

Index Numbers Construction

- **Question**:
- In the table on previous slide, how much is the percentage change in variable between years
  - 2 and 3 → $\frac{\text{Index yr3}}{\text{Index yr2}} - 1 = \frac{110}{102} - 1$ → % Δ in yr 3 over yr 2
  - 4 and 5 → $\frac{120}{116} - 1$

How to select base year?

- Generally, avoid year with extreme events / fluctuations. → 2008/09 2020/21
- Base year may have to be reset if index moves substantially away from 100.

- Suppose in the table on previous slide, base year is changed to year 4, how will the values of year 1 and year 5 be adjusted?

	yr 1	10	$\frac{10}{11.6} \times 100$
100 ←	yr 4	11.6	100
	yr 5	12	$\frac{12}{11.6} \times 100$

Index Numbers

Price relative = P_t / P_o

Price relatives

- Price relative is the price in the current time (P_t) divided by price in some base time (P_0)
- P_0 may have to be adjusted for some capital changes such as rights issue
- Taking this idea of price relative, an index may be created.

Simple Arithmetic index

- This index is created by **adding current prices** of all constituents of index and dividing the sum by sum of **all prices in the base year**.

$$I_t = B \times \frac{\sum_{j=1}^{n} P_{jt}}{\sum_{j=1}^{n} P_{j0}}$$

$$\text{Index} = \boxed{\begin{matrix}100 \\ \text{Base Value}\end{matrix}} \times \frac{\text{Sum of Prices in Current yr}}{\text{Sum of Prices in base yr}}$$

where: I_t is the value of the index at time (t);

B is the base value of the index, usually set to be 100; and

P_{jt} is the price of constituent (j) at time (t).

Price relatives

- Price relative is the price in the current time (P_t) divided by price in some base time (P_0)
- P_0 may have to be adjusted for some capital changes such as rights issue
- Taking this idea of price relative, an index may be created.

Suppose in 2018 Values of stocks X, Y & Z which make an index were 50, 60 & 105 respectively.

In 2022, X, Y & Z are valued at 72, 94 & 140 respectively. What is the index as per Simple Arithmetic index method? Index = $100 \times \frac{72+94+140}{50+60+105}$

Index Numbers

Simple Arithmetic index: Example

Index : A & B

$\frac{108+95}{100+100} \times 100$

A SIMPLE ARITHMETIC INDEX FOR STOCKS A AND B

Time period	Share price of firm A	Share price of firm B	Value of index
0	100	100	100.00
1	108	95	101.50
2	110	97	103.50
3	105	102	103.50
4	107	99	103.00
5	110	0	55.00

base

$\frac{100+100}{100+100} \times 100$ B B

$\frac{110+97}{200} \times 100$

$\frac{110+0}{200} \times 100$

- We may want to rebase the index to 100 after few years. In such cases, we divided the value of index each year by value of index in the year for which we want to rebase the index. Then multiply by 100.

Index Numbers

$\frac{y1}{y2} \times 100 : \frac{103.27}{105.97} \times 100$

Simple Arithmetic index: Example

$$\text{Value of rebased index} = \frac{I_t}{I_{RB}} \times 100$$

where: I_t is the value of the index (I) at time (t);

I_{RB} is the value of the index at the time at which we wish it to be rebased (RB).

Assume index value in Yr 1 was 103.27.

Index value in Y2 was 105.97.

Change the base to Y2. What is the value of index in Yr 1?

A REBASED SIMPLE ARITHMETIC INDEX FOR STOCKS A AND B

Time period	Index
0	(100.00 ÷ 103.50) × 100 = 96.62
1	(101.50 ÷ 103.50) × 100 = 98.07
2	(103.50 ÷ 103.50) × 100 = 100.00
3	(103.50 ÷ 103.50) × 100 = 100.00
4	(103.00 ÷ 103.50) × 100 = 99.52
5	(55.00 ÷ 103.50) × 100 = 53.14

multiply

Constructing Geometric Indices

- This index is created by multiplying all price relatives and then taking the 'n'th root of the product.

$$I_t = B \times \left[\frac{P_{1t}}{P_{10}} \times \frac{P_{2t}}{P_{20}} \times \frac{P_{3t}}{P_{30}} \times \frac{P_{4t}}{P_{40}} \times \ldots\ldots \frac{P_{nt}}{P_{n0}} \right]^{1/n}$$

$$\frac{P_{1\,t}}{P_{1\,0}} \times \frac{P_2\,t}{P_2\,0} \quad t \rightarrow \text{current} \quad 0 \rightarrow \text{base}$$

1 stock

where: B is the base value of the index;

P_{1t} is the price of constituent 1 at time (t); and

n represents the number of constituents.

Index Numbers

x y
500 100
2 10

Constructing Geometric Indices

Slide 94

A GEOMETRIC INDEX FOR STOCKS A AND B

Time period	Firm A	Firm B	SAI	GI
0	100	100	100.00	100.00
1	108	95	101.50	101.29
2	110	97	103.50	103.30
3	105	102	103.50	103.49
4	107	99	103.00	102.92
5	110	0	$\boxed{55.00}$	$\boxed{0.00}$

Year 1 : $\left[\frac{P_{A1}}{P_{A0}} \times \frac{P_{B1}}{P_{B0}}\right]^{1/2} \times 100$ $\left(\frac{108}{100} \times \frac{95}{100}\right)^{1/2} \times 100$

Index Numbers

Equally weighted index

A B C D

equal amt of $

Arithmetic (SAI) Vs Geometric index (GI)

- SAI considers price-weighted portfolio unlike GI. Hence GI is zero when one of the constituents becomes zero.
- GIs are less sensitive to large changes in asset values.
- When capital changes occur for index securities, only base price of security needs to be adjusted in case of GI.

  share capital increasing

- In case of SAI, the divisor of index constituents needs to be changed in case capital changes.
- GI should not be used for portfolio performance evaluation since GI will always underestimate the performance of constituents.
- GI does not reflect to an achievable portfolio and its value will always be less than equally weighted index.

Index Numbers

most wibely used

Market capitalization : Market Cap

Market Value Weighted indices and free float

- Market value weighted indices are created in order to avoid the problems of equally weighted indices as follows:
  - Equally weighted indices give same weight to a lower valued stock and higher valued stock. As a result, too much weight is allocated to low market capitalization stock and too less weight is given to stocks with high market capitalization.
  - Hence, equally weighted indices do not reflect the changes in value of market accurately

Price x No. of shares

- Market value weighted index is calculated as:

	A	B	
	500	100	→ Price
	x 2	x 10	→ # of shares
	1000	1000	

$$I_t = B \times \frac{\sum_{j=1}^{n} N_{jt} P_{jt}}{\sum_{j=1}^{n} N_{j0} P_{j0}}$$

B is the base value of the index;

P_{jt} is the price of constituent (j) at time (t); and

N_{jt} represents the number of shares in constituent (j) at time (t).

Index Numbers

Total shares issued by company

Market Value Weighted indices and free float
Example of market cap weighted index

- Assume that there are two stocks in an index: A (900 shares outstanding at t = 0) and B (100 shares outstanding at t = 0)

MARKET CAP WEIGHTED INDEX FOR STOCKS A AND B

Time period	Share price of firm A	Share price of firm B	MVWI
0	100	100	100.00
1	108	95	106.70
2	110	97	108.70
3	105	102	104.70
4	107	99	106.20
5	110	0	99.00

P_A S_A P_B S_B → yr 1

$$100 \times \frac{108 \times 900 + 95 \times 100}{100 \times 900 + 100 \times 100}$$

P_A S_A P_B S_B

yr → 0

$$\frac{107 \times 900 + 99 \times 100}{100 \times 900 + 100 \times 100} \times 100$$

300 → mgt ← A → Full 900 | Free Float 600

60 → Govt. ← B → 100 | 40

Free Float Market Cap Vs. Full Market Cap

- While calculating full market cap, all shares are used whether active or inactive.

available for trading

- In case of free float, only active shares i.e. shares that are available for sale and purchase in the market are used and shares held by company, promotors, government are excluded.
- Most of world's major indices have adopted 'free float' method.
- Float factor is proportion of shares that are free i.e. available for trading.

Full Market Cap Calculation	Free Float Market Cap Calculation 80%
Share Price * All outstanding shares	Share Price * All Outstanding shares * Free Float

- Free float method more rationally reflects market trends → reflects market trends better
- If free float is large volatility in market price is expected to be less and hence large institutions prefer to trade in stocks having greater free float

Index Numbers

Key global bond and equity market indices

- Various applications of indices are already discussed.
- Indices also help in comparing performances of different asset classes.

Stock market indices

FT30

- Oldest UK index
- Constructed as equally weighted geometric mean of prices of 30 stocks
- Value of 100 in 1935
- Not widely used now

FTSE All-Share

- Covers 98% (Apr 22) of market capitalization by including around 603 largest UK companies
- Based on market cap weighted

Index Numbers

Stock market indices

FTSE 100

- Covers 70% of market capitalization by including around 100 largest UK companies
- Base year Dec 1983; base value 1000
- Was introduced mainly as a basis of dealing in equity index options and futures which are derivative products

FTSE 250

- Covers 250 companies
- Market cap weighted index

- **Self read Pages 46 and 47 till Bond market indices**

Index Numbers

Bond market indices

- Most equity indices use prices and not total returns. Though total returns versions are also available for many indices.
- Bond returns reflect total returns i.e. coupons, price changes and coupon reinvestments.
- Total return indices assume that all distributions are reinvested, hence could reflect performance more accurately compared to simple price index.

FTSE Actuaries UK Gilts Indices

- Comprise 6 indices for UK Govt Bonds and 5 indices for Index-linked securities
- UK govt. indices are as per following maturity categories:
  - All bonds; up to 5 years; between 5 to 15 years; over 15 years and over 25 years

- Similar maturities exist for Index-linked securities

- Both yield and price based indices are available
- Yield indices are useful in comparing rates of return between alternative investments such as between equity and govt. bonds

Index Numbers

Bond market indices
FTSE World Govt Bond Index

- Includes categories such as
  - Govt / Corporate
  - US Govt
  - Mortgage backed securities
  - Asset backed securities
  - Emerging Americas
  - Global
  - Eurobond
  - Municipal