Investment Performance Measurement

Topics

- Total Return and its components
- Money-weighted and Time-weighted returns
- Choosing a benchmark, comparisons with investment objectives and indices
- Performance Measurement including risk-adjusted returns

Total Return and Its Components

Total Return and its Components

Learning outcomes……..

- **Identify** the components of total return for a fixed income or equity portfolio
- **Calculate** the income, capital and total return over a single period for an equity or fixed income portfolio
- **Calculate** the reinvestment return on income over a specified investment horizon
- **Explain** how returns are typically decomposed and attributed within equities (e.g. sector/stock/interaction effect) and fixed income (e.g. shift/twist/spread return)

Total Return and its Components

Total return

- Two components of total return:
  - income (from dividends or coupon) and capital value changes (gains or losses)
- Total Return from the portfolio:

$$R_p = \frac{MV_E - MV_B}{MV_B}$$

Where: MV_B is the market value of the portfolio at the beginning of the period; and

MV_E is the market value of the portfolio (including reinvested dividends or coupon payments) at the end of the period.

Total Return and its Components

Holding Period return

$$R_p = \frac{(MV_E - MV_B) + I}{MV_B} \times 100\%$$

Example

Julie purchases 1,000 shares for 704p. Six months later, just after a 10p dividend per share was received, she sells the shares for 745p. The six-month holding period return is:

$$\text{Return} = \frac{(745 - 704) + 10}{704} \times 100\% = 0.07244 \times 100\% = 7.244\%$$

Total Return and its Components

Reinvestment return

- Terminal Value of Dividends $TV = D * (1 + R)^n$
- Reinvestment return = TV – D

Example

Ethan receives a coupon of £100. The coupon is reinvested at an annual rate of 5.5% over a two-year period. The reinvestment return can be calculated as:

$$TV = £100 \times (1 + 0.055)^2 = £100 \times 1.113$$

$$= £111.30$$

The reinvestment return is £111.30 – £100 = £11.30

Total Return and its Components

Decomposing return

- Returns in excess of benchmark can be separated into following sub-components:
  - **Asset Allocation effect**: Contribution of **tactical asset allocation** to total relative performance.
  - **Currency Effect**: It is a part of allocation effect, if any, due to currency management versus the reference benchmark currency.
  - **Security Selection Effect**: Contribution of the manager's security selection ability to the total relative performance.
  - **Interaction Effect**: Combined impact of security selection and asset allocation decisions within the asset classes.

Total Return and its Components

Decomposing return: Bonds

- Bond Returns can be decomposed into
  - Maturity, sector and credit quality effects.
- Total return for a bond is composed into
  - Income component: The coupons
  - Unknown price effect: This is due to unanticipated changes in interest rates, a sector/quality change and a residual effect
    - Changes in interest rates that change the shape of yield curve are called **yield curve twists**.

Total Return and its Components

Decomposing return: Bonds

- Income effect is a return if the yield curve remains unchanged.
- Interest rate effect is due to Treasury yield curve shifts.
- Sector / Quality effect reflects the changing yield as sector (e.g. corporate) yields or credit quality changes.

Total return – yield to maturity effect – interest rate effect – sector/quality effect = residual

A consistently large positive residual would indicate superior bond selection skills over and above the effect of interest rate changes and sector/quality effects.

Total Return and its Components

Decomposing return: Equities

- Active return can be decomposed into
  - Return due to asset allocation (sector percentage)
  - Stock selection
  - Interaction effect: Combined effect of asset allocation and security selection i.e. choice of sector weights and specific stock selection

The total excess return relative to the benchmark = total asset allocation effect + total stock selection effect + total interaction effect.

Money-weighted and Time-weighted Returns

Money-weighted and Time-weighted returns

Learning outcomes……..

- **Distinguish** between money-weighted and time-weighted return, and **identify** when each method is most appropriate

- **Calculate** and **interpret** the money-weighted or time-weighted return from data provided

Money-weighted and Time-weighted returns

Calculating returns in case of regular withdrawals or deposits

Example

A pension fund has a market value of £120 million at the beginning of a month. Just before the end of this month, the trustees deposit a further £6m after the sale of a property held by the pension fund. At the end of the month, the equity portfolio is worth £123m. Therefore, the market value return on the portfolio is:

$$R_p = \frac{£123 - £120}{£120} \times 100\% = 2.5\%$$

Money-weighted and Time-weighted returns

Calculating returns in case of regular withdrawals or deposits

This would be equivalent to an annualised return of $(1.025)^{12} - 1 = 34.49\%$, but this is a misleading performance figure, because the increase of the fund by £6m was not due to the skill of the fund manager. A more accurate picture of the fund's performance would be gained by subtracting the £6m from the £123m total:

$$R_p = \frac{£123 - £6 - £120}{£120} \times 100\% = -2.5\%$$

Therefore, the funds under the fund manager's control have actually decreased in value.

Money-weighted and Time-weighted returns

Money-Weighted Rate of Return: MWRR

$$MV_0(1+r)+\sum_{t=1}^{1} C_t(1+r)^{1-t}=MV_1$$

- MWRR is IRR of a portfolio.
- It considers cash inflows and outflows during the life of investment.

Where: MV_0 is the market value of the portfolio at the beginning of the measurement period;

r is money-weighted or IRR;

C_t is a cash deposit ($-C_t$ is a cash withdrawal at time t);

$1-t$ refers to the proportion of the measurement period, which has still to elapse when the cash deposit or withdrawal occurs; and

MV_1 is the value of the portfolio at the end of the period.

Money-weighted and Time-weighted returns

Money-Weighted Rate of Return: MWRR

- MWRR is useful for comparing investment performance over time.
- However, timing and size of cash flows affect the rate of return.
- Suppose funds are added
  - and the fund does well, it overstates the return.
  - And fund does badly, it understates the return.

Money-weighted and Time-weighted returns

Money-Weighted Rate of Return: MWRR

Example

The same pension fund as the earlier example is now worth £123m at the beginning of a new month. The trustees deposit a further £5m into the fund 15 days after the beginning of the month. At the end of the month, the fund is worth £129.26m. The total return on the fund can be calculated as follows:

$$\text{Total return} = \frac{(£129.26 - £123)}{£123} \times 100\% = 5.09\%$$

The money-weighted rate of return (r) of the fund for this month is:

$$£123\text{m}(1 + r)^1 + £5\text{m}(1 + r)^{(1-0.5)} = £129.26\text{m}$$

Note that since the payment is made halfway through the month, $t = 0.5$.

Substituting values into this expression by trial and error gives us a value for r of 1%:

$r = 0.01$ or 1% per month.

Money-weighted and Time-weighted returns

Money-weighted and Time-weighted returns

Time-Weighted Rate of Return: TWRR

- TWRR gives equal weight to the returns achieved in sub-period of a particular time period.
- It is not affected by cash inflows and outflows.
- However, we need value of portfolio each time a deposit or withdrawal is made.

The **time-weighted rate of return (r)** is calculated as:

$$r = \frac{MV_{t1}}{MV_0} \times \frac{MV_{t2}}{MV_{t1} + C_{t1}} \times \ldots \times \frac{MV_1}{MV_{tn} + C_{tn}} - 1$$

Money-weighted and Time-weighted returns

Time-Weighted Rate of Return: TWRR

- Thus, in TWRR holding period return is calculated for each sub-period and then these HPRs are combined as follows:

The money-weighted return from the previous example was 1% per month. Now, suppose that immediately before £5m is paid into the fund it is still worth £123m. The rate of return achieved in the first half of the month (the first sub-period) is 0%, while the rate of return achieved in the second half of the month is approximately ((129.26 – 128) ÷ 128) = 0.98%. The **time-weighted rate of return (r)** is given as:

$$R = (1 + r_1) \times (1 + r_2) - 1$$

$$r = \{(1 + 0) \times (1 + 0.0098)\} - 1 = 0.0098, \text{ or } 0.98\%$$

Money-weighted and Time-weighted returns

Time-Weighted Rate of Return: TWRR

- In the previous example, MWRR>TWRR, and therefore it inflates the fund manager's performance.

- MWRR is higher partly due to client action, i.e. timing decision of client. It does not affect TWRR because TWRR is not affected by cash inflows and outflows as already stated.

- Hence, TWRR is preferred as a measure of portfolio return. It is particularly useful for fund managers since they do not have control over timing of cash inflows and outflows.

Money-weighted and Time-weighted returns

MWRR and TWRR

- MWRR is useful in measuring returns for asset class such as private equity where the GPs initiate capital calls from LPs i.e. they control the timing of cash flows.

- MWRR can also help to assess personal performance relative to individual financial plans and projections.

- Hence, in general TWRR is preferred as a measure of portfolio return for liquid asset classes such as equity and fixed income, whereas MWRR is preferred in case of illiquid assets that are held for reasonably long time.

Money-weighted and Time-weighted returns

Choosing a benchmark, comparisons with investment objectives and indices

Choosing a benchmark, comparisons with investment objectives and indices

Learning outcomes……..

- **Identify** the desirable properties and characteristics of an appropriate benchmark
- **Identify** the key types of benchmark used in the investment management industry
- **Explain** how to construct a benchmark portfolio

Choosing a benchmark, comparisons with investment objectives and indices

Forming a benchmark portfolio

- Investors are interested in knowing not only the absolute return a fund manager has earned but also how it compares with
  - other fund managers
  - market as a whole
  - for that sector
- Hence, we need to select appropriate benchmark by considering
  - overall performance goals
  - risk tolerance

Choosing a benchmark, comparisons with investment objectives and indices

Forming a benchmark portfolio

- Apart from the return targets, even liability-based targets are considered.
- Stock market indices are common benchmarks used, but care must be taken to use the appropriate benchmark.
- For example, a mid-cap fund return should not be compared with FTSE 100 which is a large cap index.

Indices are typically used to construct benchmark portfolios, and the clients determine the composition of the benchmark portfolio that the fund manager will be expected to match or beat.

The asset mix of the benchmark will partly reflect the clients' preferences for risk, which will in turn be determined by the size and timing of the fund's liabilities. An appropriate benchmark portfolio represents a feasible alternative to the fund manager's own portfolio.

Choosing a benchmark, comparisons with investment objectives and indices

Criteria for constructing an appropriate benchmark

- Benchmark should meet the following criteria to be a meaningful performance tool.
  - Specified in Advance
    - Benchmark should be chosen before the performance period.
  - Appropriate in terms of risk comparability
    - It should be comparable with style of the fund manager
  - Measurable
    - Some indices may not be easily measured (such as property indices)

Choosing a benchmark, comparisons with investment objectives and indices

Criteria for constructing an appropriate benchmark

- Benchmark should meet the following criteria to be a meaningful performance tool.
  - Unambiguous and transparent
    - Names and weights of securities should be clearly defined.
  - Investable
    - Fund manager should be able to fully invest into the index, if desired.
  - Availability of historical data
  - Low turnover
    - Index constituents should not keep changing frequently

Choosing a benchmark, comparisons with investment objectives and indices

Different types of investment benchmark

- Broad market indices
  - These are most widely known such as S&P 500 and MSCI World.
  - They can be domestic or international and can also consist of other asset classes.
  - However, they can be too wide for many fund managers.
- Style
  - These represent some style such as small-cap or growth etc.
  - They are relatively narrow in scope.

Choosing a benchmark, comparisons with investment objectives and indices

Different types of investment benchmark

- Absolute
  - This is a simple return objective, for example a minimum return required.
  - There is no passive investible index available to it.
- For absolute return investors, market neutral strategies are appealing being low risk or low correlation and may offer additional return with diversity.
- There are two reasons why benchmarks may still be useful for absolute return investors:
  - They provide an element of risk control
  - They help establish market neutral strategies that are uncorrelated with equity markets.

Choosing a benchmark, comparisons with investment objectives and indices

Different types of investment benchmark

- Peer Group Comparisons
  - Performance can be compared to closest competitor or again the median manager.
  - Peer group comparison can not be considered a good benchmark because the median manager is not known in advance or closest competitor may follow a different strategy.
- Customized
  - This benchmark is customized to investor’s requirement. It could be suitable for investors with more complex objectives, constraints and strategies.

Choosing a benchmark, comparisons with investment objectives and indices

Different types of investment benchmark

Use of index benchmark

A fund manager manages an equity portfolio on behalf of the trustees of a company pension scheme. The fund has an initial value of £400m and no further payments or withdrawals are made during the year.

The trustees set a benchmark portfolio which has 80% allocated to UK equity and 10% each in Japanese and US equity. The asset categories are assigned the FTSE 100, the S&P 500 and the Nikkei 225.

The value of the portfolio at the end of the year is £433m. Over the same period, the FTSE 100 and the S&P 500 have risen by 8% and 13% respectively, and the Nikkei 225 has fallen by 5%. A £400m investment in these indices in the benchmark proportions would give a benchmark value, BM, of:

$$BM = \{(£400 \times 0.8) \times 1.08\} + \{(£400 \times 0.1) \times 1.13\} + \{(£400 \times 0.1) \times 0.95\} = £428.8m$$

The fund manager has therefore outperformed the benchmark, achieving a return of 8.25%, whereas the benchmark rose by only 7.2%.

Choosing a benchmark, comparisons with investment objectives and indices

Performance Attribution

- The performance of a fund can be decomposed into i) asset allocation and ii) asset or stock selection by constructing a benchmark portfolio.

- Example:

Suppose that a pension fund which consists of gilts and UK equities is worth £400m at the beginning of a year. The benchmark portfolio consists of 60% gilts and 40% equity, where the respective indices for these asset categories are the FTSE Actuaries UK Conventional Gilts All Stocks Index and the FTSE 250 respectively.

At the beginning of the year, the portfolio has the same asset mix and proportions as its benchmark portfolio. However, exactly halfway through the year the fund manager decides to rearrange the proportions so that they hold an equal amount of equity and gilts for the rest of the year. In the first half of the year, the gilt and equity indices rise by 5% and 9% respectively, and by 8% and 3% respectively in the second half of the year. The value of the portfolio is £453m at the end of the year. The value of the benchmark portfolio at the end of the year is given as:

Performance Attribution

- The fund has a value of £453m at the end of the year.
- Value of benchmark portfolio at the end of year 1:

Value of benchmark 1

$= [0.6 \times £400m \times (1.05)(1.08)] + [0.4 \times £400m \times (1.09)(1.03)]$

$= £451{,}792{,}000$

So, the fund manager outperformed the benchmark by £1,208,000. To determine how much of this was due to the asset allocation decision, we calculate the end period value of the second benchmark portfolio:

Choosing a benchmark, comparisons with investment objectives and indices

Performance Attribution

Value of benchmark 2 (after first half of year)

$= [0.6 \times £400m \times (1.05)] + [0.4 \times £400m \times (1.09)]$

$= £426,400,000$

Value of benchmark 2 (after second half of year)

$= [0.5 \times £426.4m \times (1.08)] + [0.5 \times £426.4m \times (1.03)]$

$= £449,852,000$

Performance Attribution

Since benchmark 2 has a smaller value than benchmark 1 (£449,852,000 – £451,792,000), –£1,940,000 of the value of the portfolio is due to asset allocation decisions. Therefore, asset allocation decisions led to a decline in the portfolio's value relative to the benchmark indices. The amount of the portfolio value gained due to superior stock selection (i.e. superior to simply holding the constituents of the indices) is £3,148,000 (£453,000,000 – £449,852,000).

The fund manager in this example has shown good judgment with respect to their stock selection, but poor judgment with respect to their asset allocation decision.

Choosing a benchmark, comparisons with investment objectives and indices

Performance Measurement including Risk-adjusted Returns

Performance measurement including risk-adjusted returns

Learning outcomes……..

- **Explain** the importance of risk analysis in performance evaluation
- **Calculate** and **Interpret** the following risk-adjusted measures of return: the Sharpe measure, the Treynor measure, the information ratio and Jensen's alpha
- **Explain** how total return can be decomposed into the following: risk-free return, return due to choice of benchmark, return due to market timing, return due to diversifiable risk and pure selectivity

Performance measurement including risk-adjusted returns

Risk adjusted measures of Portfolio Performance: Sharpe Ratio

$$\text{The Sharpe measure} = \frac{R_p - R_f}{SD_p}$$

Where: R_p is the return achieved on the portfolio;

R_f is the return available from a risk-free investment; and

SD_p is the standard deviation of the return on the portfolio.

- Higher the Sharpe ratio, the better
- However, negative Sharpe ratio is misleading and not useful

Performance measurement including risk-adjusted returns

Risk adjusted measures of Portfolio Performance: Sharpe Ratio

Example

Consider two portfolios, A and B, with the following characteristics:

$R(A) - R_f = -5\%$ and $R(B) - R_f = -5\%$.

Standard deviation (A) = 20% and standard deviation (B) = 25%

The following Sharpe ratios are then obtained for portfolio A and B:

$$\text{Sharpe (A)} = \frac{(R(A) - R_f)}{\text{standard deviation (A)}} = \frac{-5}{20} = -0.25$$

$$\text{Sharpe (B)} = \frac{(R(B) - R_f)}{\text{standard deviation (B)}} = \frac{-5}{25} = -0.20$$

Since both Sharpe ratios are negative numbers, and –0.25 is lower than –0.20, one may conclude that Sharpe (A) < Sharpe (B). This suggests that portfolio B has outperformed portfolio A on the basis of higher Sharpe ratio, even though portfolio B has the larger volatility and the same return as portfolio A. This is clearly wrong.

Performance measurement including risk-adjusted returns

Risk adjusted measures of Portfolio Performance: Sharpe Ratio

Example

Over one year, two fund managers have both achieved a return of 10% on their respective portfolios. Fund manager A's portfolio had a standard deviation of 8%, while fund manager B's portfolio had a standard deviation of 9%. The return on a risk-free rate investment was 4%. The Sharpe measure for the portfolios is:

$$S_A = \frac{10 - 4}{8} = 0.75$$

$$S_B = \frac{10 - 4}{9} = 0.67$$

Fund manager A's portfolio has a higher Sharpe measure, and therefore represented better value than fund manager B's portfolio.

Performance measurement including risk-adjusted returns

Risk adjusted measures of Portfolio Performance: Treynor Ratio

$$\text{The Treynor measure} = \frac{R_p - R_f}{\beta_p}$$

Where: R_p is the return achieved on the portfolio;

R_f is the return available from a risk-free investment; and

β_p is the CAPM beta of the portfolio.

- Higher the Treynor ratio, the better
- If the portfolio is well diversified, Treynor ratio is preferred measure of performance, in case of portfolios that are not well diversified, Sharpe ratio is better.

Performance measurement including risk-adjusted returns

Risk adjusted measures of Portfolio Performance: Treynor Ratio

Example

Consider the performance of the two fund managers from the previous example. The portfolio of manager A had a beta of 0.9, indicating that the systematic risk in their portfolio was, on average, less than the risk in the market. The beta of fund manager B's portfolio was 1.5, indicating that their portfolio contained more systematic risk than the risk in the market. The Treynor measures (T_A and T_B) of each fund's performance are given as:

$$T_A = \frac{10 - 4}{0.9} = 6.67$$

$$T_B = \frac{10 - 4}{1.5} = 4.00$$

According to the Treynor measure, portfolio A gave the best 'value for money', or highest risk-adjusted return over the period.

Performance measurement including risk-adjusted returns

Risk adjusted measures of Portfolio Performance: Information Ratio

- Information ratio compares the performance of the fund relative to the risk taken to achieve the performance.

$$R = \frac{R_p - R_b}{SD_{surplus}}$$

Where: R_p is the return on the portfolio;

R_b is the return on the benchmark index; and

$SD_{surplus}$ is the standard deviation of $R_p - R_b$ over time.

- $SD_{surplus}$ is tracking error.

Performance measurement including risk-adjusted returns

Risk adjusted measures of Portfolio Performance: Information Ratio

Example

Over the last three years, using monthly data, the return on a fund has been 4.5% a year. The benchmark return over this period was 6%, and the standard deviation of the difference was 4%. The information ratio for the fund is:

$$T_A = \frac{4.5 - 6}{4} = -0.375$$

This negative figure indicates that the manager has failed to beat the index. This ratio could be compared to other funds, or compared across time to see if the manager has improved or deteriorated.

Performance measurement including risk-adjusted returns

Risk adjusted measures of Portfolio Performance: Jensen's Alpha

- Jensen's Alpha is the excess return relative to that expected / required by CAPM.
- If the fund manager can create a superior return, i.e. positive alpha, it means he possesses superior skills of fund management.

$$\text{Jensen's alpha} = J = R_p - R_b$$

$$R_b = R_f + \beta_p \{R_m - R_f\}$$

Where: R_p is the return on the fund;

R_b is the return from the benchmark portfolio; and

β_p is the benchmark's CAPM beta.

Performance measurement including risk-adjusted returns

Risk adjusted measures of Portfolio Performance: Jensen's Alpha

Example

The trustees of a pension fund decide that they want their equity portfolio to have a CAPM beta of 0.75. They instruct their fund manager to at least match the performance of an equity portfolio with a CAPM beta of 0.75 over the next year.

The risk-free rate of return over this period was 4% and the return on the market was 16%. The fund manager actually achieved a return of 15% over this year with a portfolio which had the required CAPM beta of 0.75. If the CAPM is correctly specified, a well-diversified portfolio with a beta of 0.75 should have achieved a return, R_b, of:

$$R_b = 0.04 + 0.75\,\{0.16 - 0.04\} = 0.13, \text{ or } 13\%$$

Jensen's alpha can be calculated as:

$$R_p - R_b = 0.15 - 0.13 = 0.02, \text{ or } 2\%$$

Performance measurement including risk-adjusted returns

Risk adjusted measures of Portfolio Performance: Jensen's Alpha

- Alpha is used more widely to refer to manager's skill in beating a benchmark.
- Instead of CAPM return, other factor models can also be used to calculated expected / required return.

Performance measurement including risk-adjusted returns

Risk adjusted measures of Portfolio Performance: Bond Portfolio

- Similar to Treynor ratio for equities, relative measure of performance for a bond portfolio can be calculated as follows:

$$P = \frac{r_p - r_f}{D_p / D_m}$$

- Here D_p and D_m are the duration measures for the portfolio and overall bond market, respectively.

Performance measurement including risk-adjusted returns

Risk adjusted measures of Portfolio Performance: Bond Portfolio

Example

Portfolio Y has achieved a return of 10% over the previous year, with an average duration of 16 years. Over the same period, portfolio Z only achieved a return of 8%, but with an average duration of eight years. The duration of the market over this period was eight years, while the risk-free rate was 5%. To assess the relative performance of both portfolios, the duration-adjusted excess returns for both portfolios can be calculated as follows:

$$\text{Portfolio Y} = \frac{10-5}{16/8} = 2.5$$

$$\text{Portfolio Z} = \frac{8-5}{8/8} = 3.0$$

Therefore, on a duration-adjusted basis, portfolio Z outperformed portfolio Y.

Performance measurement including risk-adjusted returns

Decomposing Risk adjusted Returns

- Jensen's Alpha is not appropriate measure of risk-adjusted performance if the fund manager has taken a lot of specific risk, since Jensen's Alpha only considers systematic risk.

- In such cases, it is better to calculate return expected from the benchmark portfolio that has same beta as the total risk of portfolio.

Performance measurement including risk-adjusted returns

Decomposing Risk adjusted Returns

- **Example:** Continuing with the example on Jensen's alpha, we have following information:
  - Portfolio P and Benchmark B both have beta of 0.75.
  - Total risk of Portfolio P, SD_p = 0.30, suppose SD_m = 0.29, beta of a well-diversified portfolio with same total risk as the portfolio can be calculated as:
  - $SD_p = \beta_b SD_m$; $0.30 = \beta_b * 0.29$; $\beta_b = 1.034$
  - Hence, we should calculate CAPM return on more appropriate benchmark portfolio with beta of 1.034; Return on more appropriate portfolio = 0.04 + 1.035 * (0.16 – 0.04) = 0.1641.
  - Thus, fund manager actually underperformed this new benchmark. This means to increase the return the fund manager resorted to stock picking but added more return without achieving required increase in return.

Performance measurement including risk-adjusted returns

Time and Risk-adjusted returns

- Risk measures calculated so far consider average risk in the portfolio over a period of time.
- However, if the total risk and beta have changed considerably during the year, then average risk can be misleading.
- Hence once possible way for assessing risk compared to timing, is to calculate risk measure on a rolling basis.

Performance measurement including risk-adjusted returns

Return due to choice of benchmark

- Performance attribution aims to identify the causes of deviations between the return of the benchmark and actual portfolio.
- The framework given by Brinson and Fachler is based on top-down investment process.
- The analysis is based on four different portfolios:
  - I. The benchmark portfolio
  - II. The stock-selected portfolio
  - III. The timing (or asset allocation) portfolio
  - IV. The actual portfolio

Performance measurement including risk-adjusted returns

Return due to choice of benchmark

- I. The return on Portfolio I is the result of investing the portfolio exactly as per strategic asset allocation and the benchmarks for the asset classes.

$$R(I) = \sum w_i^P R_i^P$$

Where: w_i^P is the strategic weight of asset class i; and

R_i^P is the return of the benchmark for asset class i.

Performance measurement including risk-adjusted returns

Return due to choice of benchmark

- II. The return on Portfolio II is due to the stock selection decision, ignoring tactical asset allocation decision.

- In the following equation, R_i^a is the actual return in asset class i.

$$R(II) = \sum w_i^P R_i^a$$

- III. The return on Portfolio III is due to the tactical asset allocation decision, ignoring stock selection decision

- The asset class weights in the following equation are actual asset class weights in the portfolio.

$$R(III) = \sum w_i^a R_i^P$$

Performance measurement including risk-adjusted returns

Return due to choice of benchmark

- IV. The return on Portfolio IV is the actual portfolio return.

$$R(IV) = \sum w_i^a R_i^a$$

- The difference between return on Portfolio I and III is '**timing effect**', and represents additional return due to asset allocation.
- The difference between return on Portfolio I and II is '**selection effect**', and represents additional return due to stock selection.
- **Interaction effect**: Sum of returns of portfolios I and IV less sum of returns of portfolios II and III. Some practitioners allocate this effect equally to timing and selection effect.

Performance measurement including risk-adjusted returns

Return due to choice of benchmark

COMPONENTS OF THE ATTRIBUTION MODEL

	Source	Calculation
Timing	R(III) – R(I)	$\sum (w_i^a - w_i^p) R_i^p$
Selection	R(II) – R(I)	$\sum (R_i^a - R_i^p) w_i^p$
Interaction	R(I) – R(II) – R(III) – R(IV)	$\sum w_i^p (R_i^p - R_i^a) + w_i^a (R_i^a - R_i^p)$
Total contribution	R(IV) – R(I)	$R_a - R_p$

Performance measurement including risk-adjusted returns

Return due to choice of benchmark

Suppose, for example, that an investor utilises a strategic asset allocation of 50% bonds, 20% domestic stocks and 30% foreign stocks. Furthermore, assume that the actual allocation, the actual returns and the benchmark returns are as presented in Table 17.2.

ACTUAL PORTFOLIO WEIGHTS, BENCHMARK RETURNS AND BENCHMARK PORTFOLIO WEIGHTS

	Actual allocation	Return benchmark	Return actual portfolio
Bond	30%	8%	7%
Domestic stocks	20%	12%	15%
Foreign stocks	50%	24%	22%

Performance measurement including risk-adjusted returns

Return due to choice of benchmark

OUTCOMES OF THE ATTRIBUTION MODEL

Portfolio	Return	Source	Contribution
I	13.6%	Timing	3.2%
II	13.1%	Selection	−0.5%
III	16.8%	Interaction	−0.2%
IV	16.1%	Total contribution	2.5%

Performance measurement including risk-adjusted returns

Case Study